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SCALING IN EXCLUSIVE ELECTROPRODUCTION AND COMPOSITE HADRONS*

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Abstract: We note that for composite hadrons the usual high-energy behaviour of amplitudes and large q^2 behaviour of form factors suggest that the amplitude for electroproduction of a single particle behaves in the Bjorken limit as

$$M \sim F_0(q^2)\, g(\omega, t)$$

for large ω. The form factor $F_0(q^2)$ gives the electromagnetic coupling of the produced hadron to the lowest resonance on the leading t-channel trajectory. We suggest that this behaviour may hold for all ω, as is the case in several models. Some implications of this behaviour for the resonance saturation of M are discussed. The general discussion is illustrated by the study of a simple dual resonance model.

1. INTRODUCTION

It has recently been suggested [1] on the basis of the dominance of leading singularities in operator product expansions near the light cone that the invariant amplitudes for the exclusive electroproduction reaction (see fig. 1),

$$\gamma(q) + h(p_1) \to h(p_2) + h(p_3) \tag{1.1}$$

behave in the Bjorken scaling limit as

$$M(s, t, q^2) \underset{\substack{q^2 \to -\infty \\ \omega, t \text{ fixed}}}{\sim} (-q^2)^d g(\omega, t)\,, \tag{1.2}$$

where

$$s/q^2 \approx 1 - \omega \qquad (1 \leq \omega \leq \infty)\,. \tag{1.3}$$

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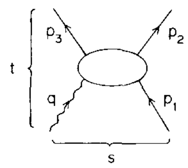

Fig. 1. Kinematics for exclusive electroproduction.

The remarkable feature of (1.2) is the factorization of the dependence on the asymptotic variable q^2 from the dependence on the scaling variable ω and the fixed momentum transfer t *.

Unfortunately, it cannot generally be shown that the leading light cone singularity dominates for processes like (1.1) where there is only one highly virtual particle [2]. Nonetheless, because of the remarkable nature of (1.2) it would be very interesting to know if it is expected to hold on some other grounds. In sect. 2 we note that such a behaviour is indeed expected for large ω if the hadrons are composite objects lying on Regge trajectories and the amplitude M is sufficiently smooth. Specifically, we find

$$M(s,t,q^2) \underset{\substack{q^2 \to -\infty \\ \omega, t \text{ fixed}}}{\sim} F_0(q^2) g(\omega, t) , \qquad (1.4)$$

where $F_0(q^2)$ is the (asymptotic form of) the form factor for transition from particle 3 (the particle produced in the direction of the virtual photon) to the spin zero particle lying on the leading Regge trajectory in the t-channel. If factorization holds for all ω and t, this argument then specifies the precise dependence on q^2 in (1.2), i.e. the asymptotic form of $F_0(q^2)$. We also comment on several calculations of the scaling limit for M in specific models and note that none are inconsistent with (1.4). It appears, then, that the scaling behaviour (1.4) is very likely to hold for physical amplitudes.

In sect. 3 we discuss some aspects of local resonance saturation of the scaling function (1.4). We are led to conjecture that $g(\omega, t)$ approaches a constant (or perhaps diverges) at threshold ($\omega = 1$) rather than vanishing. We also note that exclusive electroproduction, virtual Compton scattering (inclusive electroproduction), and two-to-two hadronic scattering are a very interesting set of reactions to study together. The assumption of local resonance saturation of their total energy discontinuities and the usual scaling behaviours for the first two reactions gives a set of consistency conditions on the resonance spectrum and resonance form factors (see fig. 2). We point out a few obvious consequences of these conditions but they clearly merit a much more thorough study.

* The quantity d is related to the light cone singularity of a certain operator product. If the hadrons are composite it is not expected to be determined by naive dimensionality considerations and thus should be regarded as a free parameter. However, since the same light cone singularity is encountered in a similar analysis of form factors, M might be expected to fall off in q^2 at the same rate as some hadronic form factor.

The more general discussions of sects. 2 and 3 are illustrated by a simple dual resonance model in sect. 4

Two general remarks should be made. First, all the discussion here applies only to the non-diffractive contributions to (1.1). For example, if the Pomeron is not an ordinary Regge trajectory, one need not expect the rapid decrease in q^2 implied by (1.4) *. Also, according to usual duality ideas, resonances would not be expected to dominate the scaling behaviour. Second, we neglect complications due to the spin of the virtual photons (and external hadrons) since there seems to be no reason to believe they are essential in our discussion.

2. COMPOSITENESS AND SCALING FOR LARGE ω

In a world of composite hadrons we expect M to be Regge behaved:

$$M(s,t,q^2) \underset{\substack{s\to\infty \\ q^2,\, t \text{ fixed}}}{\sim} F_{\alpha(t)}(q^2)(-s)^{\alpha(t)}\beta(t). \tag{2.1}$$

The form factor coupling the particle 3 to the Reggeon of spin $\alpha(t)$, $F_{\alpha(t)}(q^2)$, is expected to have a very simple dependence on q^2 as $q^2 \to -\infty$:

$$F_{\alpha(t)}(q^2) \underset{q^2\to-\infty}{\sim} F_0(q^2)(-q^2)^{-\alpha(t)}, \tag{2.2}$$

where $F_0(q^2)$ is (the asymptotic form of) the form factor coupling the spin zero particle lying on the trajectory to particle 3. The more rapid fall-off for higher spin particles exhibited by (2.2) has been derived in field theory models for composite particles [4] but is expected to be quite general since, roughly speaking, it is a manifestation of a plausible behaviour of the bound-state wave function at the origin due to centrifugal barrier effects.

Combining (2.1) and (2.2) we have

$$M(s,t,q^2) \underset{\substack{s\to\infty \\ q^2\to-\infty \\ t \text{ fixed}}}{\sim} F_0(q^2)\left(\frac{s}{-q^2}\right)^{\alpha(t)}\beta(t). \tag{2.3}$$

The scaling behaviour (1.4) will then follow for large ω if we assume M is sufficiently smooth so that there is a finite region of s and q^2 where the asymptotic forms obtained in the Regge limit ($s\to\infty$ then $q^2\to-\infty$) and the scaling limit ($q^2\to-\infty$ then $\omega\to\infty$) both hold **. Our only justification for

* Cross sections with diffractive contributions like $\gamma + N \to \rho^0 + N$ conceivably could thus be constant in q^2 as suggested by Lee [3].

** For example, if (2.1) holds uniformly in ω, i.e. for $s \gg C|q^2|$ and (2.2) holds for $|q^2| \gg N$, then (1.4) will hold for $q^2 \gg N$ and $\omega \gg C$.

this assumption, other than simplicity and naivete, is that it appears consistent with all the data for inclusive electroproduction.

The philosophy here is essentially the same as that of Abarbanel et al. [5] in their discussion of the large ω behaviour in inclusive electroproduction. The difference is that the natural behaviour (2.2) allows us to reverse the arguments and justify scaling behaviour rather than assuming it.

Scaling in exclusive electroproduction has been investigated in several models. The behaviour (1.4) has been shown to hold for all ω in ϕ^3 perturbation theory for the sum of t-channel ladder graphs [6]. In this case $F_0(q^2) \sim (q^2)^{-1}$ since hadron 3 was treated as non-composite. In all models of this type, the scaling behaviour for large ω is easily understood since they satisfy (2.2) (ref. [4]).

Exclusive electroproduction has also been studied in the parton model [7]. However, we regard the results of this calculation as inconclusive due to the large number of unknown parameters involved although the author was led to conjecture a form different from (1.4).

It appears that one needs an explicit theory of final-state interactions in order to draw any conclusions from the parton model. Yu has proposed and studied a model in which the partons interact with the physical hadrons through a dual resonance amplitude [8]. The results quoted in ref. [8] have an extra $(\log |q^2|)^{-1}$ as compared to (1.4); however, we believe that the coefficient has a divergent part which cancels this factor, as happens in his calculation of $F_0(q^2)$.

Finally, the simple dual resonance model discussed in sect. 4 agrees with (1.4) for all ω. Thus this scaling behaviour is suggested for all ω by a number of models as well as by the general argument for large ω.

We conclude this section by pointing out a source of uncertainty regarding the suggested form (1.4). If the hadrons are infinitely composite particles (as is expected if Regge trajectories rise linearly) then form factors are expected to fall faster than any power [9] (see also the last of refs. [4]). Very little is known theoretically about this possibility even though it appears to be the most likely one. In particular, one does not know if (2.2) holds. In a generalization of the dual model of sect. 4 to exponential form factors, (2.2) in fact does not hold [10]. Thus it is not surprising that M is found not to have a factorizable dependence on q^2 and the finite variables ω and t in the scaling limit *.

3. RESONANCE SATURATION OF THE SCALING FUNCTION

Since we have assumed above that $g(\omega, t)$ is Regge behaved for large ω, we expect that it satisfies a dispersion relation

* The assumption of a factorized form is also particularly doubtful in the parton model of ref. [7] since q^2 always occurs in the combination $q^2(\omega - 1)$. When this quantity is raised to a power it factorizes, but when it occurs in an exponential it does not.

$$g(\omega, t) = \int_1^\infty d\omega' \frac{\Delta g(\omega', t)}{\omega' - \omega} . \tag{3.1}$$

The discontinuity in $g(\omega, t)$ for $1 \leq \omega \leq \infty$ is a manifestation of the right-hand cut in s in M (for simplicity we suppress the cut for $-\infty \leq \omega \leq 0$ due to the left-hand cut in M).

In this section we assume that (the non-diffractive contribution to) Δg is well approximated locally ("saturated") by resonances when the resonance masses are large:

$$\lim_{\substack{q^2 \to -\infty \\ \alpha^{-1}(N)/q^2 \text{ fixed}}} [d(N) \operatorname{Res}_{\alpha(s)=N} M(s, t, q^2)] = F_0(q^2)\, \Delta g(\omega, t) , \tag{3.2}$$

where $1 - \omega \equiv \alpha^{-1}(N)/q^2$ and $d(N) \approx d\alpha^{-1}(N)/dN$ is the separation between resonances in s *. This assumption is motivated by the discussion of local duality for inclusive electroproduction given by Bloom and Gilman [11] (see also ref. [12]). The analog of the assumption of ref. [11] for exclusive electroproduction would be

$$\operatorname{Res}_N M(s, t, q^2) \underset{\omega = 1 - N/q^2}{\approx} F_0(q^2) \Delta g(\omega, t) \tag{3.3}$$

for q^2 sufficiently large. The basic difference between these two assumptions is that (3.2) is weaker since it requires N to be large (one must jump from one resonance to the next as q^2 increases). We prefer to assume (3.2) here since it is sufficient for most of our purposes. Furthermore dual models in general satisfy exactly only the weaker assumption (see sect. 4, for example). For particular choices of parameters these models may *approximately* satisfy (3.3), however, just as the Veneziano model for hadronic amplitudes approximately satisfies local duality for small N for particular trajectory intercepts. We regard such an approximate satisfaction of (3.3) to be a much more likely physical situation than the exact satisfaction assumed in some models (e.g. universal form factors, see ref. [13] for a review). Much stronger consequences follow, of course, from (3.3) - see the footnote following (3.7) and ref. [13] for a general discussion for inclusive electroproduction.

There are two particularly interesting assumptions that are stronger than (3.2) but weaker than (3.3):

(i) Consistency between Regge behaviour and scaling for $\omega \to \infty$,

* Resonance saturation requires infinitely rising Regge trajectories. We will assume linear trajectories for simplicity below so that $d(N) = 1$ and $\alpha^{-1}(N) \approx N$. The residue is understood to include all approximately degenerate poles near $\alpha(t) = N$.

$$\lim_{q^2 \to -\infty} \left[\lim_{\substack{N \to \infty \\ q^2 \text{ fixed}}} \mathrm{Res}_N M \right] = \lim_{\omega \to \infty} \left[\lim_{\substack{q^2 \to -\infty \\ \omega \text{ fixed}}} \mathrm{Res}_N M \right]$$

$$= \lim_{\omega \to \infty} F_0(q^2) \Delta g(\omega, t) . \qquad (3.4)$$

This is a consequence of the usual Regge-resonance duality for fixed q^2 and the smoothness assumption of sect. 2.

(ii) Consistency between asymptotic form factors and scaling for $\omega \to 1$,

$$\lim_{N \to \infty} \left[\lim_{\substack{q^2 \to -\infty \\ N \text{ fixed}}} \mathrm{Res}_N M \right] = \lim_{\omega \to 1} \left[\lim_{\substack{q^2 \to -\infty \\ \omega \text{ fixed}}} \mathrm{Res}_N M \right]$$

$$= \lim_{\omega \to 1} F_0(q^2) \Delta g(\omega, t) . \qquad (3.5)$$

Let us speculate briefly on the threshold behaviour ($\omega \to 1$) of $g(\omega, t)$. Suppose form factors are power behaved,

$$\mathrm{Res}_N M \xrightarrow[q^2 \to -\infty]{} C_N(-q^2)^{-\gamma_s(N)} , \qquad F_0(q^2) \xrightarrow[q^2 \to -\infty]{} (-q^2)^{-\gamma_t} . \qquad (3.6)$$

The leading s-channel Regge trajectory has $\gamma_s(N) = \gamma_s' + N$ from (2.2) and cannot contribute to the satisfaction of (3.5). All we can conclude is that there must be an effective γ_s independent of N and thus from (3.5) and (3.6)

$$\Delta g(\omega, t) \approx (1 - \omega)^{\gamma_s - \gamma_t} . \qquad (3.7)$$

Since *a priori* $\gamma_s - \gamma_t$ can have any value * this argument is inconclusive **. On the other hand, the integral (3.1) must diverge to give poles in t. The lowest pole clearly gives the contribution

$$g(\omega, t) \approx \frac{1}{\alpha_t} (1 - \omega)^0 . \qquad (3.8)$$

We thus conjecture that $g(\omega, t)$ approaches a constant for $\omega \to 1$ or diverges (if $\gamma_s - \gamma_t < 0$). The dual model discussed below satisfies this conjecture.

Finally we comment briefly on the consistency conditions on the resonance spectrum and residues implied by scaling. Consider the three classes of reactions shown in fig. 2, i.e. purely hadronic, exclusive electroproduction, and inclusive electroproduction. The imposition of factoriza-

* $\gamma_s - \gamma_t < 0$ does not require any physical quantity to become infinite since $(1 - \omega) \geq s_{\min}/q^2$.

** With the stronger assumption (3.2), taking $N = 0$ we find $\gamma_s = \gamma_s' = \gamma_t$ and $\Delta g(\omega, t) \approx (1 - \omega)^0$.

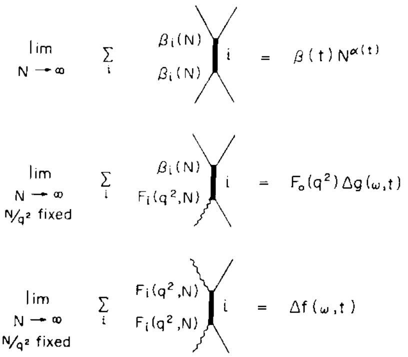

Fig. 2. Consistency conditions from resonance dominance of (a) hadronic reactions, (b) exclusive electroproduction, and (c) inclusive electroproduction. The sum i is over resonances of about the same mass N but of different spins and types.

tion as shown clearly makes the solution of this set of relations subtle *. The main problem is the consistency between the rapid decrease in (b) and the constant behaviour in (c). Clearly there must be more than one term in the sum i. There must be some states for large N whose coupling to two hadrons is very small but whose coupling to a hadron and a virtual photon is very large (to compensate the usual decrease for large q^2). Crudely speaking, the virtual photon must be a very good probe into details of the hadronic resonance structure.

4. DUAL RESONANCE MODEL

In this section we discuss scaling in exclusive electroproduction in a simple dual resonance model in order to illustrate the preceding discussion. The dual model we study is the model which expresses current amplitudes as dual n-point functions with certain arguments set equal to constants [14]. This model was chosen because it has a large number of desirable qualitative features [10, 15]: usual s-t duality, form factors with power decrease dominated by vector meson poles, (2.2) is satisfied, the power decrease of the form factors is correlated with the location of fixed poles as suggested by field theory models of composite particles, scaling holds for inclusive electroproduction.

In this model M is given by a B_5 function (see fig. 3)

$$M(s,t,q^2) = \int_0^1 \int_0^1 \mathrm{d}v_1\,\mathrm{d}v_2\; v_1^{\gamma t-1}(1-v_1)^{-\alpha(s)-1} \times v_2^{\gamma s-1}(1-v_2)^{-\alpha(t)-1}(1-v_1 v_2)^{-\alpha(q^2)+\alpha(s)+\alpha(t)} . \qquad (4.1)$$

* It is even more subtle when the q^2 dependence in (b) and (c) is given by a sum over vector meson amplitudes, each satisfying (a). Such a sum must clearly have an infinite number of terms.

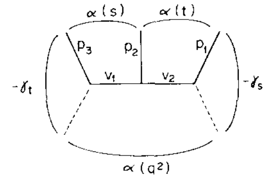

Fig. 3. Choice of variables for eq. (4.1). The dotted lines can be thought of as fictitious "leptons" which form the current.

In the Bjorken scaling limit we obtain ($v_1 = -x_1/\alpha(s)$):

$$M(s,t,q^2) \sim (-q^2)^{-\gamma_t}(1-\omega)^{-\gamma_t} \int_0^\infty dx_1 \int_0^1 dv_2$$

$$\times x_1^{\gamma_t - 1} v_2^{\gamma_s - 1}(1-v_2)^{-\alpha(t)-1} \exp\left[-x_1\left(1 + \frac{\omega}{1-\omega} v_2\right)\right]$$

$$= (-q^2)^{-\gamma_t}(1-\omega)^{-\gamma_t} \frac{\Gamma(\gamma_s)\Gamma(\gamma_t)\Gamma(-\alpha(t))}{\Gamma(\gamma_s - \alpha(t))}$$

$$\times {}_2F_1(\gamma_s, \gamma_t; -\alpha(t)+\gamma_s; -\frac{\omega}{1-\omega}) . \tag{4.2}$$

Using the standard formulae for continuation of the hypergeometric function [16], we find

$$M(s,t,q^2) \underset{\substack{\text{Bj. limit} \\ \omega \to \infty}}{\sim} (-\omega)^{\alpha(t)} \Gamma(-\alpha(t))\Gamma(\gamma_t + \alpha(t))$$

$$+ (-\omega)^{-\gamma_t} \frac{\Gamma(-\gamma_t - \alpha(t))\Gamma(\gamma_s)\Gamma(\gamma_t)}{\Gamma(\gamma_s - \gamma_t - \alpha(t))} . \tag{4.3}$$

Let us compare (4.3) with the Regge limit * (using (4.1) and ref. [16]):

$$M(s,t,q^2) \underset{s \to \infty}{\sim} \frac{\Gamma(-\alpha(q^2))\Gamma(\gamma_t + \alpha(t))\Gamma(-\alpha_t)}{\Gamma(-\alpha(q^2) + \gamma_t + \alpha(t))} (-s)^{\alpha_t}$$

$$+ \frac{\Gamma(\gamma_s)\Gamma(-\gamma_t - \alpha(t))\Gamma(\gamma_t)}{\Gamma(\gamma_s - \gamma_t - \alpha(t))} (-s)^{-\gamma_t} . \tag{4.4}$$

We find the form factor

* In addition to the Regge pole there is a "fixed pole" giving $(-s)^{-\gamma_t}$ whose location is connected with the asymptotic fall-off of form factors [10].

$$F_{\alpha(t)}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-\gamma_t - \alpha(t)} \Gamma(\gamma_t + \alpha(t)) , \tag{4.5}$$

whereas the spin zero form factor is (from (4.4) or directly from (4.1)):

$$F_0(q^2) = \frac{\Gamma(-\alpha(q^2))\,\Gamma(\gamma_t)}{\Gamma(-\alpha(q^2)+\gamma_t)} \underset{q^2 \to -\infty}{\sim} (-q^2)^{-\gamma_t} \Gamma(\gamma_t) , \tag{4.6}$$

so (2.2) is satisfied. We see therefore that M is sufficiently smooth so the limits can be interchanged and furthermore (1.4) holds to all ω with

$$g(\omega, t) = \frac{\Gamma(\gamma_s)\,\Gamma(-\alpha(t))}{\Gamma(\gamma_s - \alpha(t))} (1-\omega)^{-\gamma_t} \,{}_2F_1(\gamma_s, \gamma_t; -\alpha(t)+\gamma_s; -\frac{\omega}{1-\omega}) . \tag{4.7}$$

For future reference we note that

$$\Delta g(\omega, t) = \pi \frac{\Gamma(\gamma_s)}{\Gamma(\gamma_t)\,\Gamma(\gamma_s - \gamma_t + 1)} \omega^{-\gamma_s}(\omega - 1)^{\gamma_s - \gamma_t}$$

$$\times\, {}_2F_1(\gamma_s, \alpha(t)+1; \gamma_s - \gamma_t + 1; \frac{\omega - 1}{\omega}) , \qquad (1 \leqslant \omega \leqslant \infty) . \tag{4.8}$$

The threshold behaviour is easily obtained using [16]

$$g(\omega, t) \underset{\omega \to 1}{\sim} \frac{\Gamma(\gamma_s - \gamma_t)\,\Gamma(-\alpha(t))}{\Gamma(\gamma_s - \gamma_t - \alpha(t))} + (1-\omega)^{\gamma_s - \gamma_t} \frac{\Gamma(\gamma_s)\,\Gamma(\gamma_t - \gamma_s)}{\Gamma(\gamma_t)} . \tag{4.9}$$

Thus $g(\omega, t)$ has the threshold behaviour conjectured in sect. 3. The term with constant behaviour has the poles in t as expected.

We now study the resonance saturation of the scaling function. For simplicity we give here the results for $\alpha(t) = -1$ although the conclusions are valid for all $\alpha(t)$. We find

$$\mathrm{Res}_N\, M = \pi \frac{\Gamma(\gamma_s)\,\Gamma(\gamma_s - \gamma_t + 1 + N)\,\Gamma(-\alpha(q^2)+N)}{\Gamma(\gamma_s - \gamma_t + 1)\,\Gamma(1+N)\,\Gamma(-\alpha(q^2)+\gamma_s + N)} \tag{4.10}$$

for $\alpha(t) = -1$. Thus

$$\lim_{\substack{q^2 \to -\infty \\ N/q^2 \text{ fixed}}} \mathrm{Res}_N\, M = (-q^2)^{-\gamma_t} \omega^{-\gamma_s}(\omega - 1)^{\gamma_s - \gamma_t} \pi \frac{\Gamma(\gamma_s)}{\Gamma(\gamma_s - \gamma_t + 1)} . \tag{4.11}$$

The model therefore satisfies the weak local duality assumption (3.2) since for $\alpha(t) = -1$, eq. (4.8) gives

$$\Delta g(\omega, t) = \omega^{-\gamma_s}(\omega - 1)^{\gamma_s - \gamma_t} \pi \frac{\Gamma(\gamma_s)}{\Gamma(\gamma_t)\,\Gamma(\gamma_s - \gamma_t + 1)} . \tag{4.12}$$

It does not satisfy the stronger assumption (3.3), however, because

$$\lim_{q^2 \to -\infty} \mathrm{Res}_N\, M = (-q^2)^{-\gamma_s}\, \pi\, \frac{\Gamma(\gamma_s)\,\Gamma(\gamma_s - \gamma_t + 1 + N)}{\Gamma(\gamma_s - \gamma_t + 1)\,\Gamma(1+N)}\,, \tag{4.13}$$

which does not agree with (4.12). On the other hand, the relations (3.4) and (3.5) are satisfied since

$$\begin{aligned}\lim_{N \to \infty} [\lim_{q^2 \to -\infty} \mathrm{Res}_N M] &= (-q^2)^{-\gamma_t}\, \pi\, \frac{\Gamma(\gamma_s)}{\Gamma(\gamma_s - \gamma_t + 1)} (\omega - 1)^{\gamma_s - \gamma_t} \\ &= \lim_{\omega \to 1} F_0(q^2)\Delta g(\omega, t)\,,\end{aligned} \tag{4.14}$$

and

$$\begin{aligned}\lim_{q^2 \to -\infty} [\lim_{N \to \infty} \mathrm{Res}_N M] &= \lim_{q^2 \to -\infty} \pi\, \frac{\Gamma(\gamma_s)}{\Gamma(\gamma_s - \gamma_t + 1)} N^{-\gamma_t} \\ &= (-q^2)^{-\gamma_t}\, \pi\, \frac{\Gamma(\gamma_s)}{\Gamma(\gamma_s - \gamma_t + 1)}\, \omega^{-\gamma_t} \\ &= \lim_{\omega \to \infty} F_0(q^2)\Delta g(\omega, t)\,.\end{aligned} \tag{4.15}$$

Finally we look at the consistency conditions (fig. 2) in the model. The model does not factorize fully [17] so the conditions will not be rigorously satisfied but it is interesting to look at them anyway. The inclusive electroproduction amplitude in the model [10] satisfies (for $\alpha(t) = -1$ and $M = \nu W_2$):

$$\begin{aligned}\Delta f(\omega, t) &= \pi\omega(\omega - 1)^{2\gamma_s - 1} \frac{\Gamma(\gamma_s)\,\Gamma(\gamma_s)}{\Gamma(2\gamma_s)} \\ &= \lim_{\substack{N \to \infty \\ N/q^2 \text{ fixed}}} \mathrm{Res}_N M \\ &= \lim_{\substack{N \to \infty \\ N/q^2 \text{ fixed}}} \Bigg\{ \pi N \sum_{\substack{i,j,k=0 \\ i+j+k=N}}^{\infty} \left(\frac{\Gamma(\gamma_s)\,\Gamma(\gamma_s - \gamma_t + 1 + i)\,\Gamma(-\alpha(q^2) + i + k)}{\Gamma(\gamma_s - \gamma_t + 1)\,\Gamma(1+i)\,\Gamma(-\alpha(q^2) + \gamma_s + i + k)}\right) \\ &\quad \times \left(\frac{\Gamma(\gamma_s)\,\Gamma(\gamma_s - \gamma_t + 1 + j)\,\Gamma(-\alpha(q^2) + j + k)}{\Gamma(\gamma_s - \gamma_t + 1)\,\Gamma(1+j)\,\Gamma(-\alpha(q^2) + \gamma_s + j + k)}\right)\left(\frac{\Gamma(2\gamma_s - 2 + k)}{\Gamma(1+k)\,\Gamma(2\gamma_t - 2)}\right)\Bigg\}\,.\end{aligned} \tag{4.16}$$

The presence of several sums in (4.16) as compared to (4.10) suggests the presence of many more states coupling in case (c) of fig. 2. Indeed, the scaling function arises from terms in the sum with $k \propto N$ for which the couplings are much larger (due to the final group of gamma functions) than those in (4.10).

We believe the study of this dual model is very useful for gaining intuition about the expected behaviours of amplitudes with highly virtual photons. It would be interesting to know if one is led to similar conclusions in the more sophisticated dual model of Manassah and Matsuda [18] which also satisfies gauge invariance.

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REFERENCES

[1] Y. Frishman, V. Rittenberg, H. R. Rubenstein and S. Yankielowicz, Phys. Rev. Letters 26 (1971) 798.
[2] J. Sucher and C. H. Woo, Phys. Rev. Letters 27 (1971) 696;
W. Rühl, Kaiserslautern preprint TP-2 (1971).
[3] T. D. Lee, Columbia U. preprint NYO-1932(2)-187 (1971).
[4] M. Ciafialoni and P. Menotti, Phys. Rev. 173 (1968) 1575;
M. Ciafialoni, Phys. Rev. 176 (1968) 1898;
D. Amati, R. Jengo, H. R. Rubinstein, G. Veneziano and M. A. Virasoro, Phys. Letters 27B (1968) 38.
[5] H. D. I. Abarbanel, M. L. Goldberger and S. B. Treiman, Phys. Rev. Letters 22 (1969) 500.
[6] S. Matsuda and M. Suzuki, Phys. Rev. D1 (1970) 1778;
J. D. Dorren, Nucl. Phys. B36 (1972) 541.
[7] P. Roy, Phys. Rev. D5 (1972) 227.
[8] L.-P. Yu, Phys. Rev. D4 (1971) 2775, 3113.
[9] S. Mandelstam, Proc. of the 1966 Tokyo Summer Lectures in theoretical physics, ed. G. Takeda (W. A. Benjamin, Inc., New York, 1967).
[10] J. H. Weis, Lawrence Radiation Laboratory report UCRL-19780 (1970) (unpublished).
[11] E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25 (1970) 1140.
[12] P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B19 (1970) 432.
[13] Y. Avni and M. Milgrom, Weizmann preprint WIS 71/26 Ph (1971).
[14] H. Sugawara, Tokyo U. of Education report (1969) (unpublished);
M. Ademollo and E. DelGuidice, Nuovo Cimento 63A (1969) 639.
[15] R. C. Brower, A. Rabl and J. H. Weis, Nuovo Cimento 65A (1970) 654.
[16] Bateman Manuscript Project, Higher transcendental functions, ed. A. Erdelyi (McGraw Hill Book Co., New York, 1953), vol. 1, p. 108, 109.
[17] D. Z. Freedman, Phys. Rev. D1 (1970) 1133.
[18] J. T. Manassah and S. Matsuda, Phys. Letters 36B (1971) 229.